# *Understanding the Neutrino Mechanism of Core-Collapse Supernovae: the Antesonic Condition*

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## Abstract

The mechanism of core-collapse supernovae is unknown. Despite considerable effort, most simulations of supernovae are not successful, and it has proven difficult to revive the stalled accretion shock, particularly for more massive stellar progenitors. Although it is known that the stalled accretion shock turns into explosion when the neutrino luminosity from the collapsed core exceeds a critical value ($L^{\rm crit}$) (the "neutrino mechanism"), the physics of $L^{\rm crit}$, as well as its dependence on the properties of the proto-neutron star (PNS) and changes to the heating/cooling mechanisms has never been systematically explored. We quantify the deep connection between the solution space of steady-state accretion flows with bounding shocks and the neutrino mechanism. In particular, we show that for the simple model of pressure-less free-fall onto a shock bounding an isothermal accretion flow with sound speed $c_T$ there is a maximum, critical sound speed $c_T^{\rm crit}$ above which it is impossible to maintain accretion with a standoff shock, because the shock jump conditions cannot be satisfied. The physics of this critical sound speed is general and does not depend on a specific heating mechanism. We show that $(c_T/v_{\rm esc})^2 = 3/16 = 0.1875$ at $c_T^{\rm crit}$ – the "antesonic" condition. We generalize this result to the more complete supernova problem, where the critical condition for explosion can be written as max $(c_S/v_{\rm esc})^2 \cong 0.19$ over a broad range in accretion rate and microphysics. Other criteria proposed in the supernova literature fail to capture the physics of $L^{\rm crit}$. In addition, we explore the effects of collective neutrino oscillations on $L^{\rm crit}$.

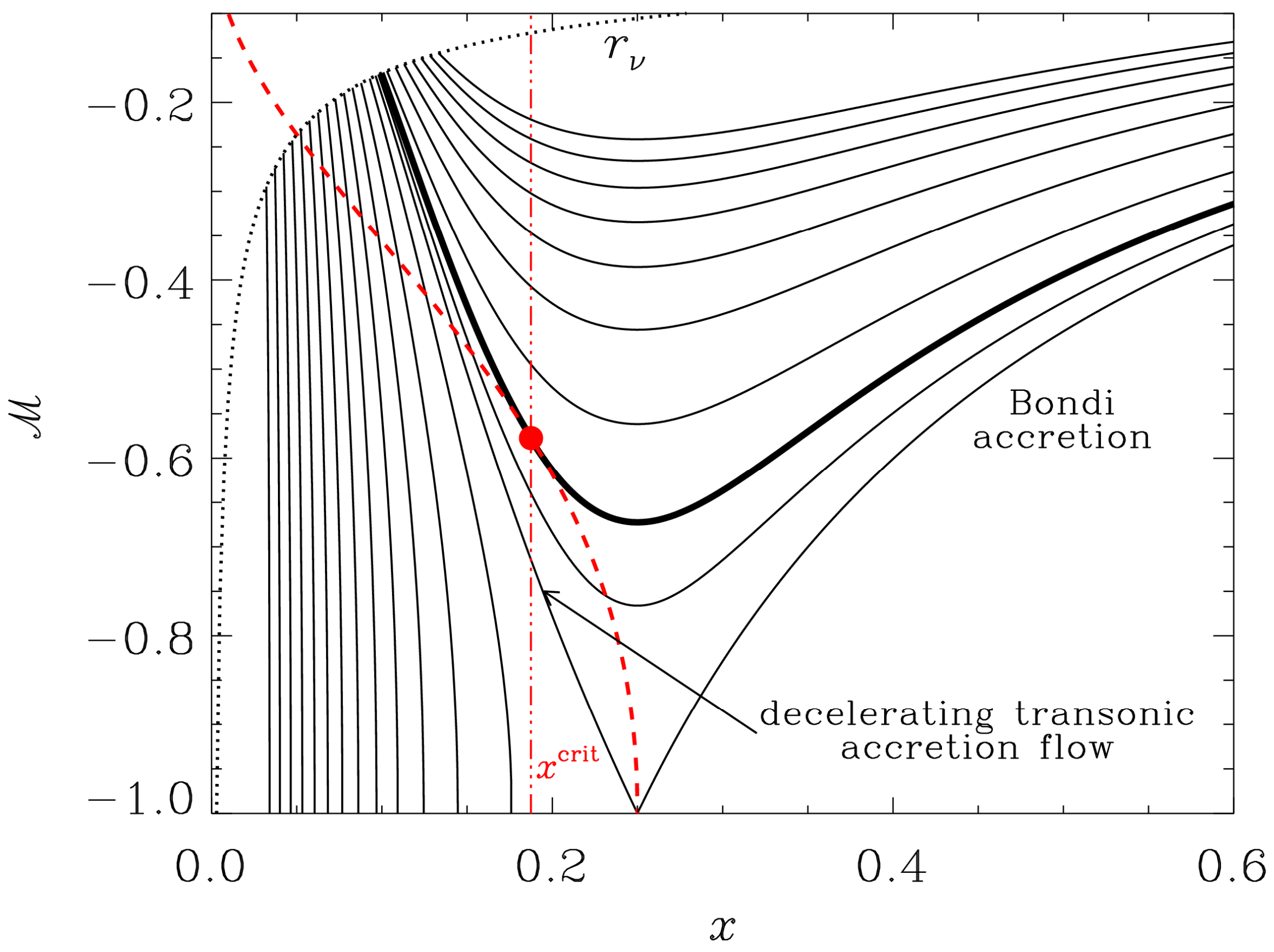

**Figure 1**: Isothermal accretion plotted in the space of Mach number $\mathscr{M}$ and rescaled radial coordinate $x = rc_T^2/(2GM)$. Solid black lines show solutions to the accretion equation with $\dot{M} = 1\ M_\odot$/s starting from $r_\nu$ = 30 km (grey dotted line). The value of $c_T^2$ increases from the black solid line starting at lowest $x$ to the highest line. The red dashed line shows velocity just downstream of a shock positioned at any $x$, assuming that the upstream flow is in pressure-less free fall A viable accretion solution with a shock starts at $r_\nu$ and follows any of the black lines until it crosses the red dashed line, where it jumps to the assumed upstream velocity profile. Above a certain $c_T^{\rm crit}$ there is no accreting solution with a steady-state shock. The critical value $x^{\rm crit}$, where this happens, is shown with a vertical red dash-dot-dot line.

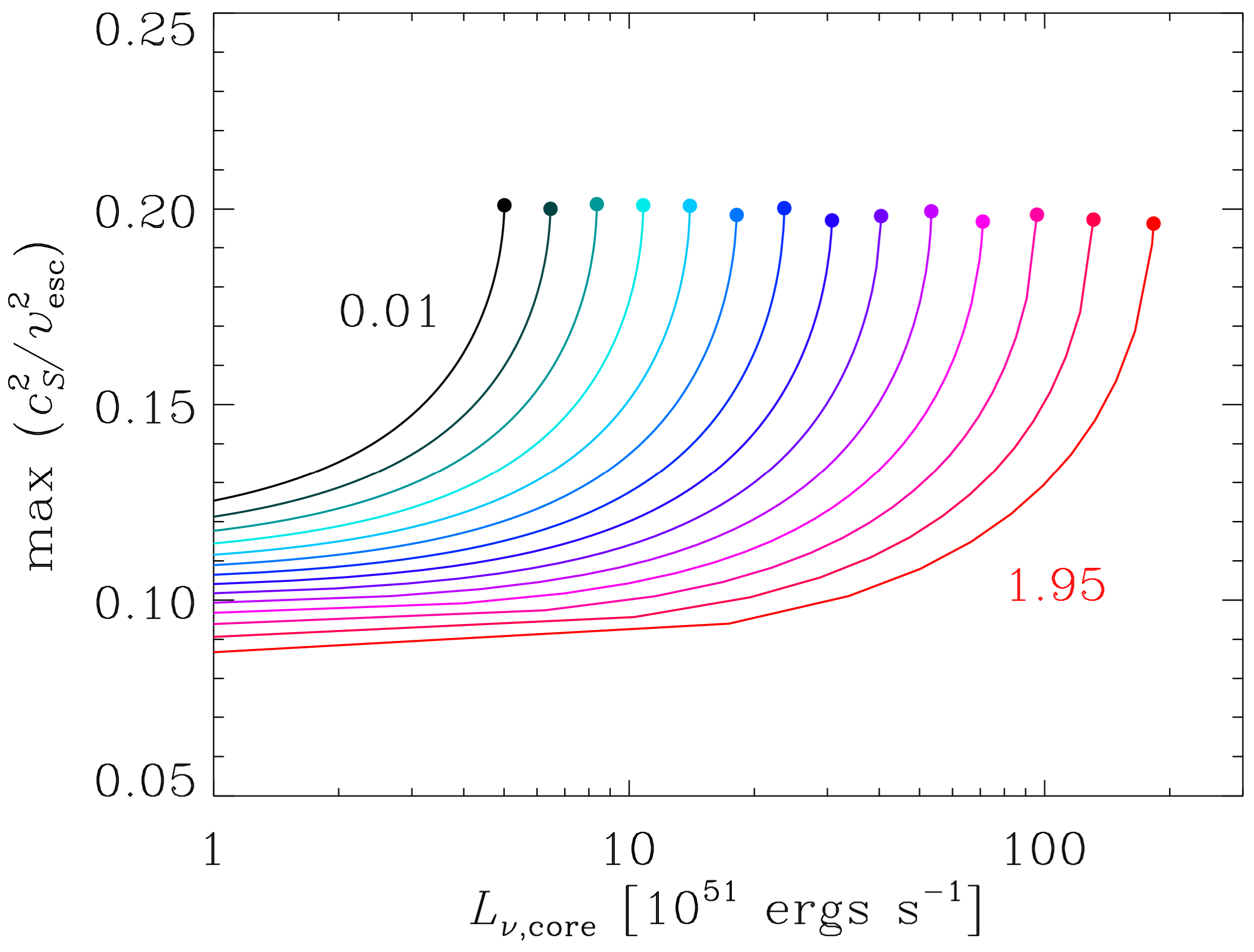

**Figure 3**: The antesonic condition. Maximum value of the ratio of the adiabatic sound speed $c_S$ to the escape velocity $v_{\rm esc}$ as a function of $L_{\nu,\rm core}$ (individual lines). The color coding corresponds to different values of $\dot{M}$ going left to right from 0.01 $M_\odot$/s (black) to 1.95 $M_\odot$/s (red). The filled circles at the end of the lines mark the value at $L^{\rm crit}$. Although the critical condition plotted here appears to be a local condition on the sound speed in the accretion flow, the quantity max $(c_S/v_{\rm esc})^2$ is merely a scalar metric for solution space of Euler equations and thus it is a global condition.

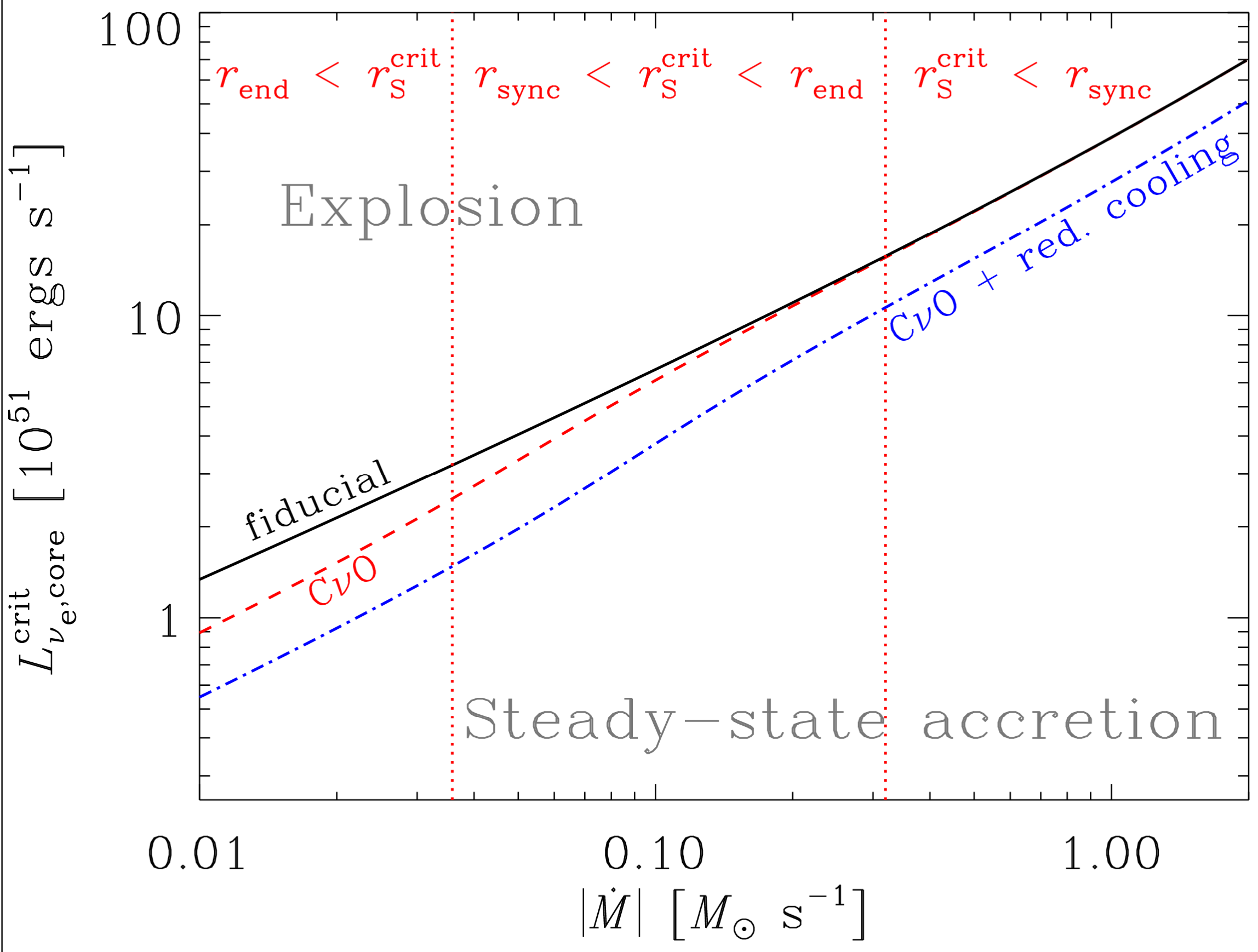

**Figure 5**: Critical core neutrino luminosity $L^{\rm crit}$ as a function of $\dot{M}$ for $M$=1.2 $M_\odot$ and $r_\nu$ = 60 km. The solid black line shows the fiducial calculation (no CνO), while the red dashed line is with CνO. The vertical red dotted lines mark different regimes of the effect. The blue dash-dotted line includes CνO and has the neutrino cooling reduced by a factor of 2 to approximate multi-dimensional effects.

## Isothermal model

Although it is known that the stalled accretion shock turns into explosion when the neutrino luminosity from the collapsed core exceeds a critical value ($L^{\rm crit}$) (the "neutrino mechanism"), the physics of $L^{\rm crit}$ has never been systematically explored. In Pejcha & Thompson (2011) we found that the idealized model problem of spherical steady-state isothermal accretion bounded by a standoff shock is the key to understanding $L^{\rm crit}$ and the explosions of supernova. The velocity structure of such flow is described by the equation

$$\left(\mathscr{M} - \frac{1}{\mathscr{M}}\right)\frac{{\rm d}\mathscr{M}}{{\rm d}x} = \frac{2}{x} - \frac{1}{2x^2},$$

where $\mathscr{M} = v/c_T$ is the Mach number and $x = rc_T^2/(2GM)$. The standing shock wave in the flow exists at a point that satisfies the two Rankine-Hugoniot shock jump conditions and we assume that the matter incoming to the shock is in pressure-less free fall.

In Figure 1, we show with black solid lines Mach-number profiles of flows with a single constant mass accretion rate $\dot{M}$ starting at a fixed radius $r_\nu$ corresponding to the surface of the star, and with a fixed velocity $v(r_\nu)$. Different lines correspond to different values of $c_T$. The essential point of Figure 1 is that for a fixed $\dot{M}$ the presence of a shock (red dashed line) is not guaranteed for all values of $c_T$. For the smallest values of $c_T$, the required shock radius – the intersection of the black solid line with red dashed line – would be below the radius of the star, an unphysical situation. As we step to higher $c_T$, the shock appears at $r_\nu$ and moves outward. At a critical value $c_T^{\rm crit}$ (thick solid line), the shock can only coincide with the sonic point at $\boldsymbol{x^{\rm crit} = c_T^2/v_{\rm esc}^2 = 3/16 = 0.1875}$ (red dash-dotted vertical line – the "antesonic" condition). A shock is not possible for $c_T > c_T^{\rm crit}$, because the red dashed line lies below the black lines in the range of radii of interest. Positions of $c_T^{\rm crit}$ as a function of $\dot{M}$ are shown in Figure 2.

In Pejcha & Thompson (2011) we showed that the physics of $c_T^{\rm crit}$ is equivalent to $L^{\rm crit}$, and that it does not depend on a specific heating mechanism.

## Antesonic condition

As $L^{\rm crit}$ cannot be determined in an time-dependent calculation, it is desirable to find an equivalent expression composed of variables within the accretion flow itself. In particular, we check whether the $x^{\rm crit} = c_T^2/v_{\rm esc}^2 \cong 0.19$ at $c_T^{\rm crit}$ holds also for $L^{\rm crit}$ in the more realistic calculations.

In order to that, we solve the 1D equations of steady-state hydrodynamics coupled to equations for the electron fraction and simple radiative transport as a two-point boundary value problem between the neutrinosphere and the bounding accretion shock. We employ an equation of state consisting of ideal and relativistic gases with a chemical potential for the electrons/positrons, and simplified heating and cooling due to the charged-current neutrino processes. The inner boundary is taken to be the proto-neutron star neutrinosphere. Here, the PNS core luminosity $L_{\nu,\rm core}$ is specified. We obtain radial dependencies of thermodynamic variables as well as the steady-state shock radius as a function of $L_{\nu,\rm core}$, $\dot{M}$, the mass of the proto-neutron star, and the neutrinosphere radius (see Pejcha & Thompson 2011).

We plot in Figure 3 how max $(c_S/v_{\rm esc})^2$ changes with $L_{\nu,\rm core}$ and $\dot{M}$. We find that the value of this parameter is surprisingly constant at 0.20 for $L_{\nu,\rm core} = L^{\rm crit}$ over almost three orders of magnitude in $\dot{M}$. Furthermore, we took 1350 calculations of $L^{\rm crit}$ over a range of $M$, $\dot{M}$, and $r_\nu$ and computed a histogram of values of max$(c_S/v_{\rm esc})^2$ at $L^{\rm crit}$. We find that the histogram can be very well fit with a Gaussian with a maximum at 0.193 and with a width of 0.009, that is only 5% of its value! This is much better consistency than any other explosion condition proposed in literature. In particular, the commonly assumed ratio of advection and heating times does not capture the physics of $L^{\rm crit}$ as can be seen in Figure 4.

## Effect of collective neutrino oscillations

Most of the heating below the shock occurs due to absorption of $\nu_e$ on neutrons and protons, while the more energetic $\nu_\mu$ and $\nu_\tau$ escape without much interaction. Thus, if the luminosities in each flavor are similar, ~2/3 of the total neutrino luminosity generated by the PNS is essentially unused. However, due to the high density of neutrinos in this region, self-interaction between neutrinos becomes important and can lead to a range of phenomena called "collective neutrino oscillations" (CνO, Pantaleone 1992). Because the neutrino heating efficiency depends on the square of neutrino energy, conversion of the more energetic $\nu_\mu$ and $\nu_\tau$ to $\nu_e$ can increase the heating rate and lower $L^{\rm crit}$.

We extend the code developed in Pejcha & Thompson (2011) to include the effect of CνO using a scheme recently reviewed by Dasgupta et al. (2011), which assumes that $\nu_e$ effectively increase their energy as a function of radius. The increase commences at $r_{\rm sync}$ and is done at $r_{\rm end}$. These radii are function of neutrino luminosities, energies, and $r_\nu$. Figure 5 shows $L^{\rm crit}$ including CνO as a function of $\dot{M}$ (red dashed line) along with the fiducial calculation (black solid line). We see that CνO lowers $L^{\rm crit}$ to ~0.65 times the fiducial value for $\dot{M} < 0.01\ M_\odot$/s. For higher $\dot{M}$, the effect is smaller. Figure 6 shows the effect of CνO for different parameter sets and compares it to other effects like radiation transport and multi-dimensional effects.

## References

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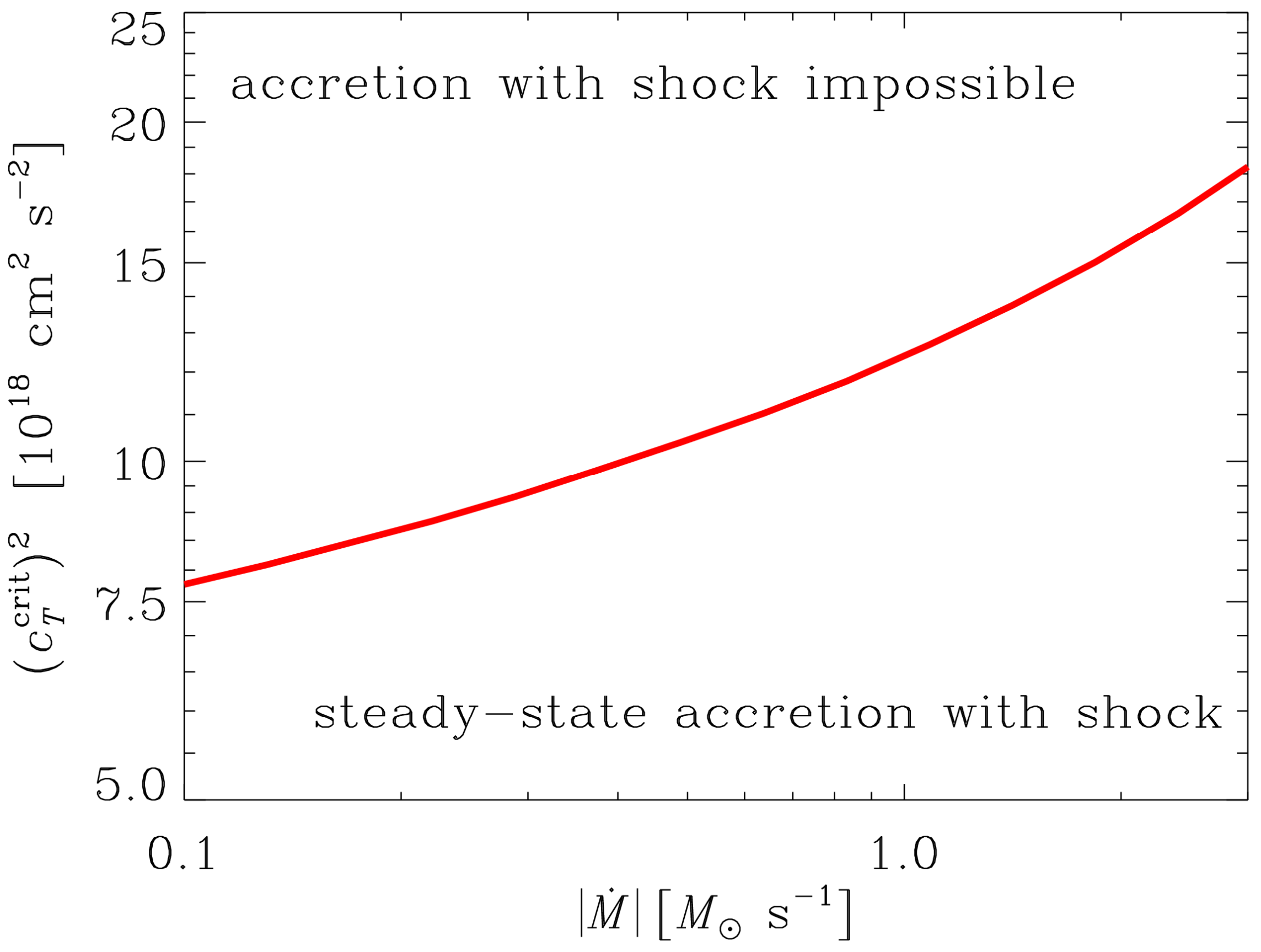

**Figure 2**: The maximum isothermal sound speed $c_T^{\rm crit}$ that allows for a shock in the flow at a given $\dot{M}$. The red critical curve separates steady-state solutions from supersonic neutrino-driven wind – supernova explosion (Burrows & Goshy 1993). The PNS parameters – $M$ and $r_\nu$ – are the same as in Figure 1.

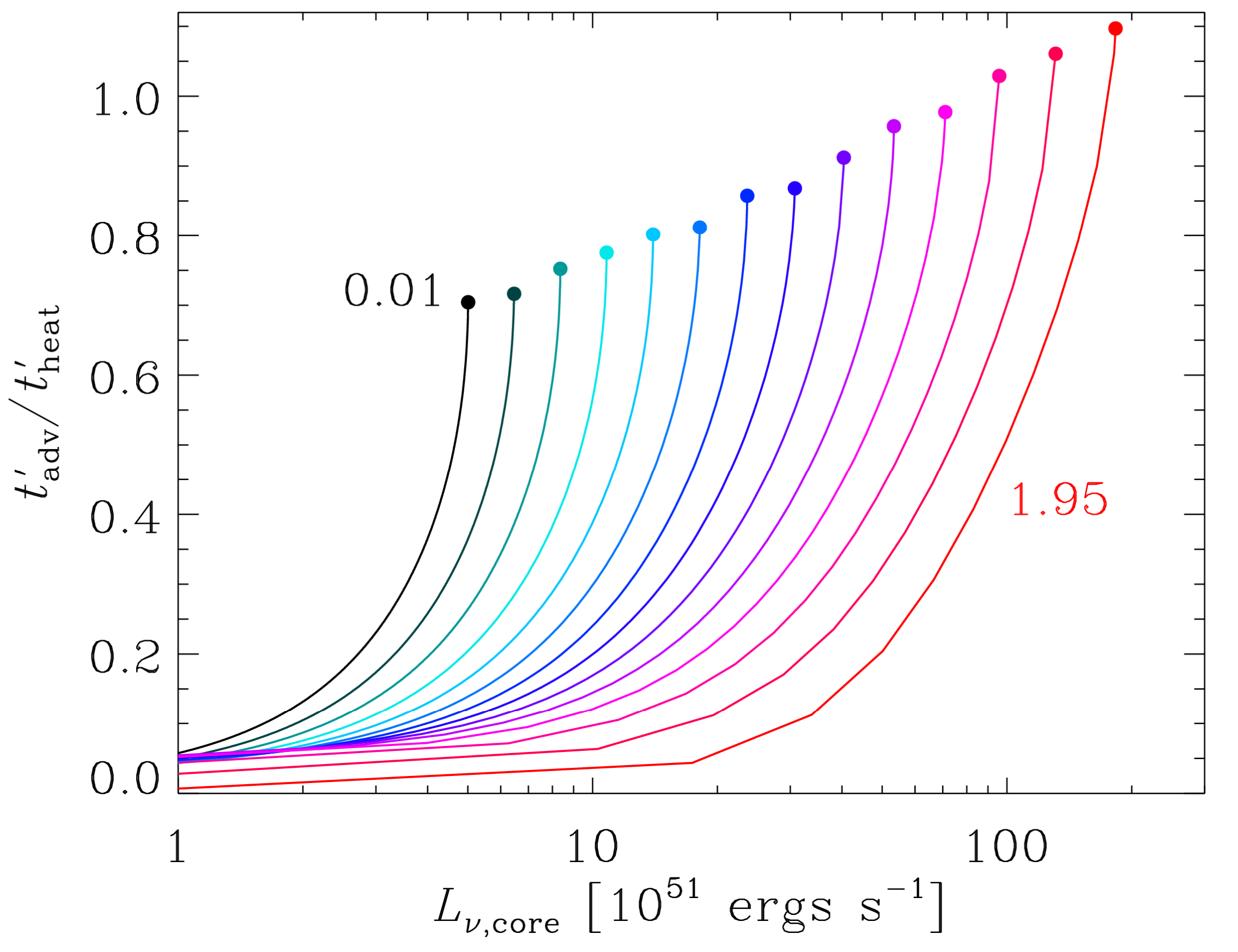

**Figure 4**: Ratio of advection and heating times as defined by Murphy & Burrows (2008). Each line shows the ratio as a function of core luminosity $L_{\nu,\rm core}$ and fixed $\dot{M}$. Color coding is the same as in Figure 3. Note that the values of the ratio of the times at $L^{\rm crit}$ (dots at the end of lines) change as a function of $\dot{M}$, unlike the antesonic ratio in Figure 3.

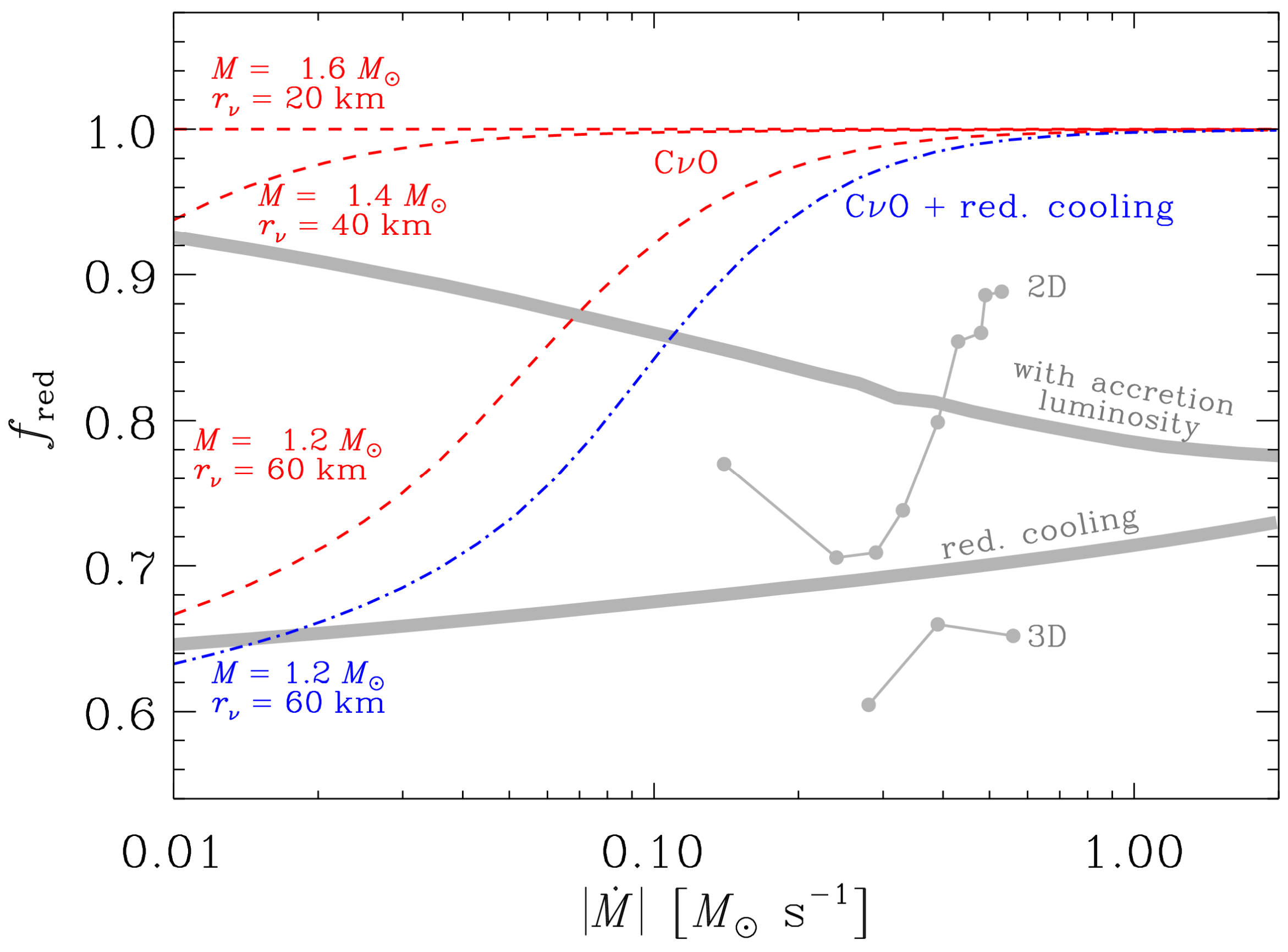

**Figure 6**: Relative reduction of $L^{\rm crit}$ with respect to the fiducial 1D calculation ($f_{\rm red}$) as a function of $\dot{M}$ including various physical effects. Red dashed lines show $f_{\rm red}$ for CνO. Lines are labeled with $M$ and $r_\nu$ of the PNS. Blue dash-dotted line shows the effect of CνO and reduced cooling rate by a factor of 2 relative to a calculation with reduced cooling only. The upper thick grey line shows $f_{\rm red}$ when neutrinos from cooling of the accretion flow are taken into account and the lower thick grey line illustrates $f_{\rm red}$ for cooling rate reduced by a factor 2 (Pejcha & Thompson 2011). The grey solid lines with points show $f_{\rm red}$ for multi-dimensional effects when the dimension of the simulation is increased from 1D to 2D or from 1D to 3D (from Nordhaus et al. 2010).