1D non-LTE time-dependent radiative transfer of supernova ejecta with CMFGEN

Luc Dessart¹ and D. John Hillier²

¹:Laboratoire d'Astrophysique de Marseille, France ²:University of Pittsburgh, USA

Long-term Goal:

- Design a radiative-transfer tool to model the interaction of radiation and matter in SN ejecta
- Limit to 1D to better focus on physics (e.g. non-LTE etc.)
- > Treat all SN types , i.e. thermonuclear or gravitational collapse
- Start from physical input models
- > Compute LC and spectra to compare with observations and constrain stellar evolution/explosion
- ➤ CMFGEN applied/tested for SNe II-pec, II-P, IIb, Ib, and Ic Dessart & Hillier (2005ab,06,08,10,11ab)
- Future work: Type Ia SNe

Synopsis

- Context
- Radiative-transfer issues: line blanketing & opacity, electron-scattering, non-LTE (HeI), time-dependence (Hα), decay energy & non-thermal processes
- Steady-state model; Dependencies
- Time-dependent models: LC and spectra for SNe II-pec, II-P, IIb, Ib, and Ic

Chronology of events in the life of a CCSN

- 1 sec: Core collapse, bounce, shock revival
- 1 min to 1 day: shock propagates and breaks out (1st EM signature). Fallback? NS vs. BH formation?
- At breakout: $E_{rad} \sim E_{kin}$; $E_{rad} >> E_{th}$; $\tau_{cont} \sim 10^6$
- Mins to days: Final ejecta acceleration to homology (V∝R)
- Ejecta properties: E_{kin}~10⁵¹erg, M_{ejecta}~ few M_☉, V_{exp}~3000km/s, M(⁵⁶Ni) ~ 0.1M_☉
- Generic subsequent Evolution controlled by

```
Cooling (Expansion & Radiative losses)
```

versus Heating (Radioactive decay & Recombination).

modulo Transport (dynamic radiative diffusion --- opacity/composition/ionization, dT/dr!)

Their variations cause the diversity of CCSN Light Curves and Spectra

- Weeks to months: Photospheric phase (τ>>1)
- After a (few) month(s): Transition to Nebular phase (τ<<1)
- 1-10ⁿ years: SNR, CSM interaction, light echoes

Modeling of Light Curves and Spectra: Some Features

- Radiative diffusion ⇒ Time-dependent Transport, Energy/opacity problem
- Strong collisional proc. at depth ⇒ LTE, blackbody SED
- Strong electron scattering ⇒ non-LTE above R_{phot}, J ≠ B, flux "dilution"
- Large radii ⇒ photospheric density is low (weak collisions) ⇒ non-LTE effects
- $\tau_{line} >> \tau_{cont}$ & Expansion \Rightarrow P-Cygni profiles
- Large # of lines and large ejecta velocity ⇒ Line overlap and blanketing
- Steep ρ and/or N_e distribution \Rightarrow Line formation, low linear polarization
- High V, low $\rho \Rightarrow t_{rec} \sim R/V \Rightarrow time-dependent effects (UC05, DH08)$
- γ-rays ⇒ High-energy e⁻ ⇒ Non-thermal effects (Lucy 1991)

Non-LTE Time-Dependent Radiative Transfer Modeling with CMFGEN Co-moving frame formulation for homogously-expanding ejecta (MKH75)

Gas

Coupling

Radiation

Rate Equation:
$$\rho \frac{D(n_i/\rho)}{Dt} = \frac{1}{r^3} \frac{D(r^3 n_i)}{Dt} = \sum_{j \neq i} (n_j R_{ji} - n_i R_{ij})$$

& charge conservation

Energy Equation:
$$\rho \frac{De}{Dt} - \frac{P}{\rho} \frac{D\rho}{Dt} = 4\pi \int_0^\infty \chi_v (J_v - S_v) dv + De_{decay}/Dt$$

where

e = internl energy/unit mass

$$= \frac{\frac{3}{2}kT(n+n_e)}{\mu m_H n} + \frac{\sum n_i E_i}{\mu m_H n}$$
 (Excitation + Ionization)

RTE 0th moment:

$$\frac{1}{cr^3} \frac{D(r^3 J_v)}{Dt} + \frac{1}{r^2} \frac{\partial (r^2 H_v)}{\partial r} - \frac{vV}{rc} \frac{\partial J_v}{\partial v} = \eta - \chi J_v$$

RTE 1st moment:

$$\frac{1}{cr^3} \frac{D(r^3 H_v)}{Dt} + \frac{1}{r^2} \frac{\partial (r^2 K_v)}{\partial r} + \frac{K_v - J_v}{r} - \frac{vV}{rc} \frac{\partial H_v}{\partial v} = -\chi H_v$$

1-D Non-LTE time-dependence using CMFGEN

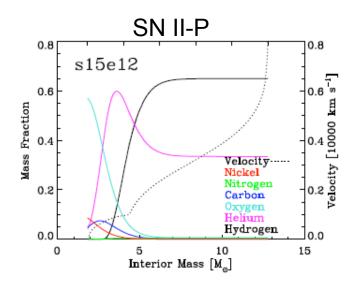
(Hillier & Miller 1998; Dessart & Hillier 2005, 2008, 2010, 2011ab)

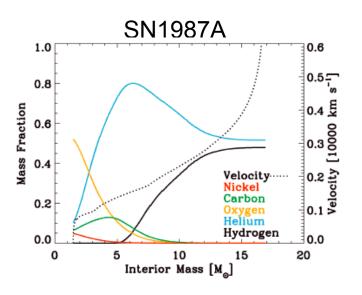
- Simultaneously solves the radiative-transfer equation in the CMF frame, the statistical-equilibrium and the energy equations; Fully implicit solver; partial linearization of SEE.
- Accurate description of I/J/H: moments of RTE with all important terms in v/c, $\partial/\partial t$, $\partial/\partial v$, $\partial/\partial \mu$, $\partial/\partial r$
- > RTE solved for at ~10⁵ frequency points (with coupling). Coverage: ~10A to ~5μm
- Detailed description of the gas: 25 species & 15 ionization stages. Non-LTE ionization
- **Large model atom**: few 10000 levels and few 100000 transitions.
- Approximation: Use of Super-Levels. Easy check on approx by switching to full levels.
- Non-LTE: All important radiative + collisional rates included explicitly.
- Non-LTE line blanketing: All continuum and line opacity sources included explicitly
- ➤ Time-dependent terms in SEE ⇒ Time-Dependent Ionization
- > Solution of Spencer-Fano equation for **non-thermal heating/ionization/excitation rates**
- Adaptive grid: ~7 pts per τ-decade at each time (asset over hydro: mass grid); $\tau \in [10^{-8} \text{ to } 10^{6}]$

1-D Non-LTE time-dependence using CMFGEN

(Hillier & Miller 1998; Dessart & Hillier 2005, 2008, 2010ab)

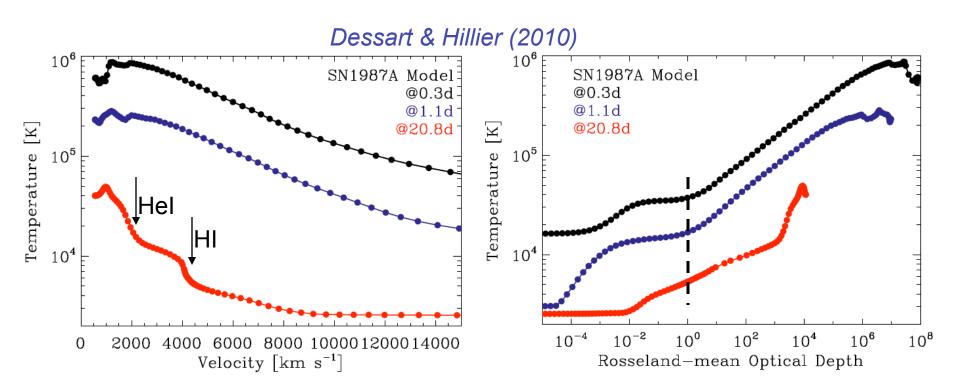
- Physical consistency: stellar-evolution + hydro input: $X_i(m)$, T(m), $\rho(m)$, v(m) r(m)
 - => use SN light to constrain pre-SN evolution and explosion
- Full-ejecta simulation, e.g. no "artificial" boundary conditions, X_i stratification
- Decay energy: Computed with Monte Carlo γ-ray transport code (local or non-local)
- Model requires ~5-10Gb, i.e. (NT=2000) x (ND=100) x (NBNDS+1=4) x 8
- Arr Time step: Δt = 0.1t => 45-50 steps to go from 0.3 to 21d, or 10 to 1000d => 3 months!





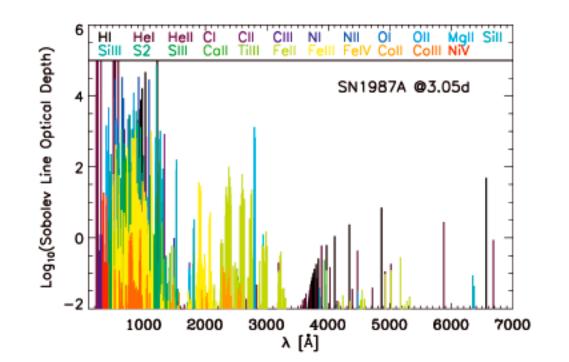
Spatial grid

- ✓ Use optical-depth scale, not mass scale (e.g. hydro code)
- ✓ Good resolution of photosphere; Eddington factors 1/3 →1
- ✓ Naturally adjusts to resolve recombination fronts
- ✓ Converged results with ~100 depth points.



Model atom --- Line Blanketing

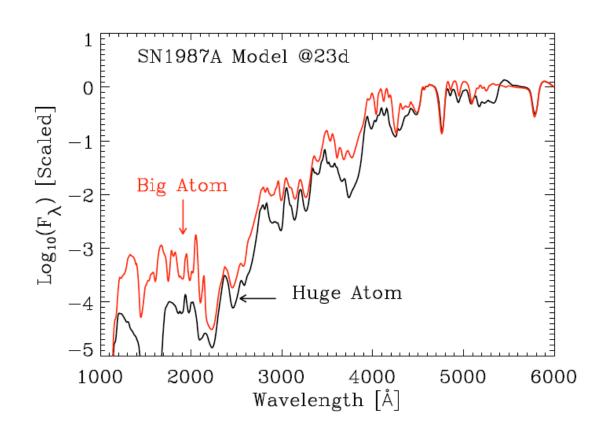
- Many species: H, He, CNO, IME, IGE
- Many ions (I-VII) treated simultaneously
- Huge model atom to account for all species/ions and sources of opacities (lines and continuum).
- Sources of atomic data: Opacity project etc. Essential.
- In non-LTE, need for all rates (radiative & collisional)
- Strong line blanketing



Species	N_f	N_s	N_{trans}	Ref	Details
Ηı	30	20	435		<i>n</i> ≤30
HeI	51	40	374		$n \leq 11\text{TRP}$
HeII	30	13	435		<i>n</i> ≤30
Cı	26	14	120		$n \leq 2s2p^{3} {}^{3}P_{0}$
CII	26	14	87		$n \leq 2s2s4d^2D_{5/2}$
Сш	112	62	891		$n \leq 2s8f^1Fo$
CIV	64	59	1446		$n \leqslant 30$
Nı	104	44	855		$n \leq 5f^2$ Fo
NII	41	23	144		$n \leq 2p^3d^1P_1$
Oı	51	19	214		$n \leq 2s^2 2p^3$ 4S 4f 3F3
OII	111	30	1157		$n \le 2s^2 2p^2 {}^{3}P 4d^2D_{5/2}$
Ош	86	50	646		$n \leq 2p^4f^1D$
OIV	72	53	835		$n \leq 2p^2p^3p^2P$
Ov	78	41	523		$n \leq 2s5f^1Fo_3$
Naı	71	22	1614		n ≤30w2W
Mg II	65	22	1452		n ≤30w2W
SiII	59	31	354		$n\leqslant 3s^2$ 1 S $7g^2$ G $_{7/2}$
SiIII	61	33	310		$n \leq 3s5g^1Ge_4$
SiIV	48	37	405		$n \leq 10f^{2}$ Fo
SII	324	56	8208		$n \leq 3s3p^3$ 5 S $4p^6$ P
SIII	98	48	837		$n \leq 3s3p^2 {}^2D 3d {}^3P$
Siv	67	27	396		$n \leq 3s3p ^{3}P 4p^{2}D_{5/2}$
Ca II	77	21	1736		$n \leq 3p^630w2W$
TiII	152	37	3134		$n \leq 3d^2$ ³ F $5p^4D_{7/2}$
Тіш	206	33	4735		$n \leq 3d^6f^3Ho_6$
Fe II	115	50	1437		$n \leq 3d^{6} ^{1}G1 4sd^{2}G_{7/2}$
Fe III	477	61	6496		n ≤3d ⁵ ⁴ F 5s ⁵ Fe ₁
Fe IV	294	51	8068		$n \leq 3d^{4} ^{5}D 4d^{4}G_{5/2}$
Fe V	191	47	3977		$n \leq 3d^{3} ^{4}F 4d^{5}Fe_{3}$
Fe VI	433	44	14103		$n \leq 3p5 ^2P 3d^4 ^1S ^2Pc_{3/2}$
Fe VII	153	29	1753		$n \leq 3p5 ^{2}P 3d^{3} b^{2}D ^{1}P_{1}$
CoII	144	34	2088		$n \leq 3d^6$ 5D 4s4p 7Do ₁
CoIII	361	37	10937		$n \leq 3d^6 ^5D 5p^4P_{3/2}$
CoIV	314	37	8684		$n \leq 3d^{5} ^{2}P ^{4}p ^{3}P_{1}$
Co V	387	32	13605		$n \leq 3d^4 {}^3F 4d^2H_{9/2}$
Co VI	323	23	9608		$n \leq 3d^{3} {}^{2}D 4d {}^{1}S_{0}$
Co VII	319	31	9096		$n \leq 3p5$ ² P d ⁴ ³ F ² D _{3/2}
Ni v	183	46	3065		$n \leq 3d^5 ^2D3 4p ^3F_3$
Ni vi	314	37	9569		$n \leq 3d^{4} ^{5}D 4d^{4}F_{9/2}$
Ni vii	308	37	9225		$n \le 3d^3 {}^2D 4d {}^3P_2^{7}$
	6426		143054		

Model atom --- Line Blanketing

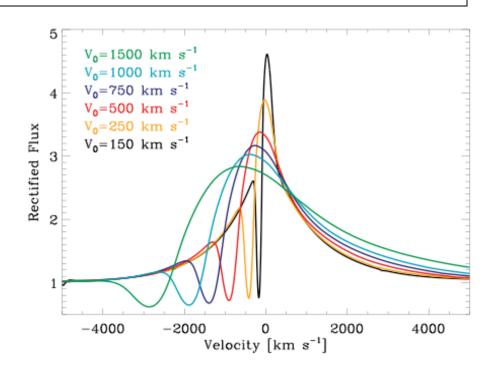
- Difficult to obtain converged results, especially in UV, UB bands
- Accuracy of atomic data?
- Illustration for importance of Fel, Nill and larger Fell model (big → huge atom)



Effects associated with Electron-scattering: Frequency redistribution

- photon scattering with free electrons causes frequency redistribution
- Non-coherent scattering in CMF caused by the thermal motion of scatterers: V_{thermal}
- Coherent scattering in CMF due to expansion, Redshift in Observer's frame: V_{expansion}
- V_{expansion} > V_{thermal} ⇒ the redshift dominates: P-Cygni profile with enhanced red-wing flux
- V_{expansion} < V_{thermal} ⇒ non-coherent redshift/blueshift dominates: Symmetric profile (SNe IIn)

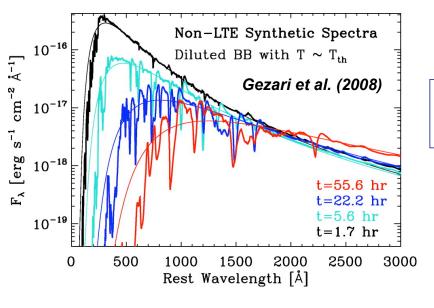
Effect of varying V_{phot} on $H\alpha$ morphology



Electron-scattering: Flux "Dilution"

Mihalas book; Dessart & Hillier (2005b)

- Scattering-dominated atmosphere => $\kappa << \sigma => \lambda = \kappa / (\kappa + \sigma) << 1$
- Eddington Approximation and dB/d τ =const. => Flux $\propto 2\sqrt{\lambda}$ B(τ =1/ $\sqrt{3}\lambda$)
 - -> τ =1/ √3 λ : thermalization depth
 - $-> 2\sqrt{\lambda}$: Factor of "dilution" (<<1)
- Introduce of ''dilution'' factor ξ : $\mathbf{F_{obs}} = (\xi \mathbf{R_{phot}}/\mathbf{D})^2 \pi \mathbf{B_v}(\mathbf{T_c})$; $(\mathbf{R_{phot}} > \xi \mathbf{R_{phot}})$

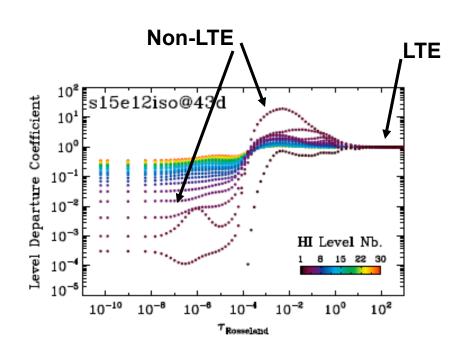


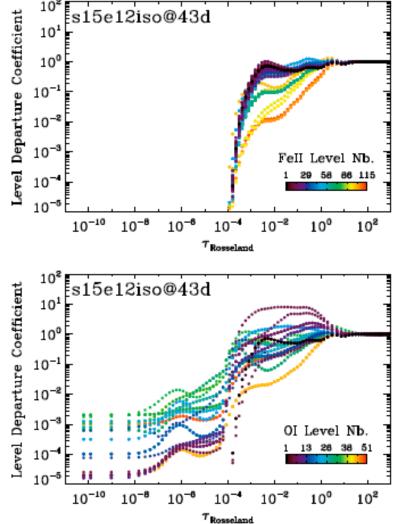
Example with early post-breakout models: SEDs fitted with $B(T_{th})$ and $\xi \sim 0.6$

- Solution of the statistical-equilibrium equations using all collisional/radiative rates. About 2000 levels/ variables => Yields non-LTE level populations, ionization, line-blanketing
- High density => Strong collision & weak scattering => LTE
- Low density => weak collision: Dominance of scattering => Drives gas out of LTE
- \triangleright Large departure coefficients in regions of τ <1 but we recover LTE at depth!

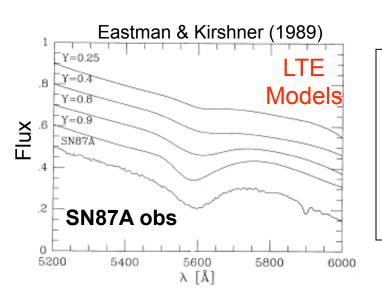
Non-LTE effects

Dessart & Hillier (2011)

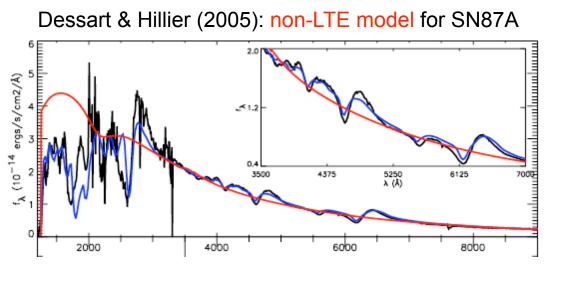


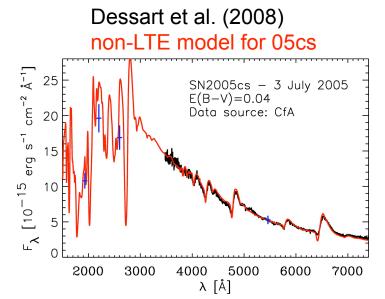


non-LTE effects: Influence on Hel lines at early times

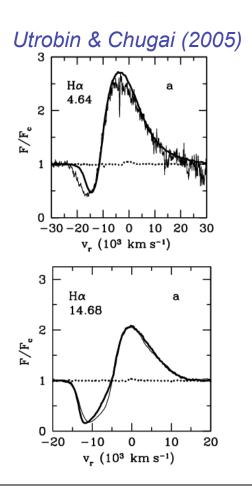


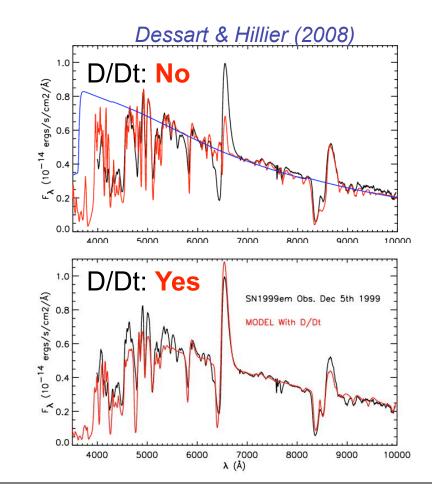
- → non-LTE: Account of radiative rates in the determinations of level populations
- → non-LTE allows good fit to Hel lines using a "standard" BSG/RSG He abundance at early times
- ⇒ non-LTE key for abundance determinations





Spectral Modelling: time-dependent effects

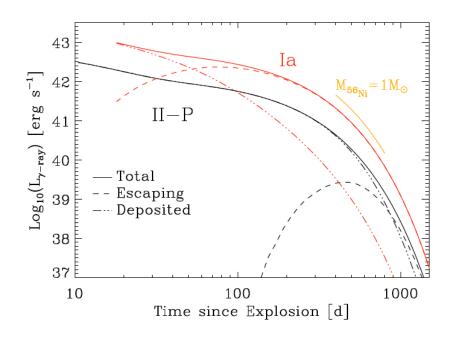


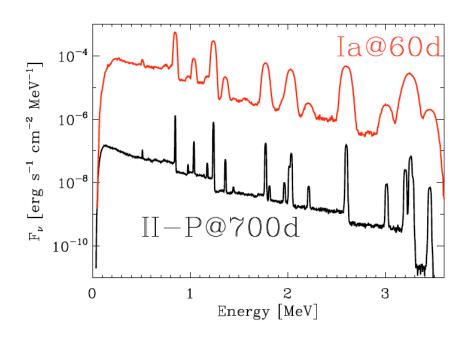


- High V, low $\rho \Rightarrow t_{rec} \sim R/V \Rightarrow$ time-dependent effects (UC05, DH08)
- Retain D/Dt terms in SSE and Energy equations
- Key to yield strong recombination lines in the absence of Lyman/Balmer flux
- $H\alpha$ strength at rec. epoch sensitive to N_e rather than ρ profile

Treatment of Energy Deposition: γ-ray Monte-Carlo transport code

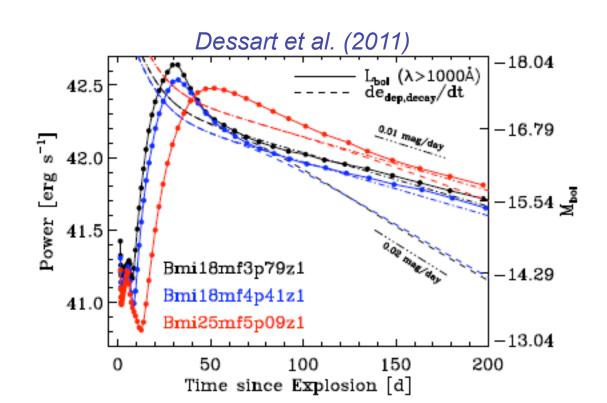
- > 2-step decay chain: $^{56}\text{Ni} \rightarrow ^{56}\text{Co} \rightarrow ^{56}\text{Fe} \ (\tau_{1/2} = 6.1 \text{ and } 77.27\text{d})$
- \triangleright β+ decay, electron capture \Rightarrow γ-rays, neutrinos, positrons, pairs
- Monte-Carlo code to follow γ-rays trough ejecta subject to Compton Scattering and photoelectric absorption
- \succ τ_{v} >>1: assume local deposition as heat. τ_{v} <1: non-local, non-thermal





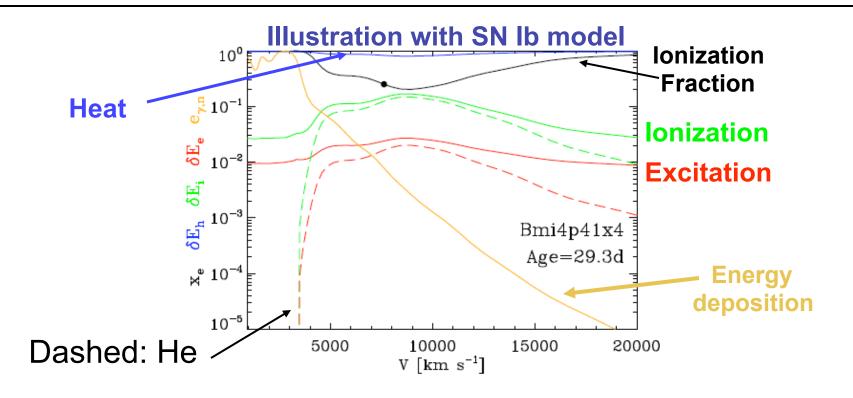
γ-ray escape and the nebular-phase decline rate

- Full γ-ray trapping in SNe II-P and II-pec up to ~700d
- γ-ray escape as early as ~50d in SNe IIb/Ib/Ic
- Key diagnostic of ejecta mass (insensitive to ionization issues)
- Fast-declining SNe IIb/Ib/Ic LCs points to low-mass progenitors



Non-thermal processes

- Solution of the Spencer-Fano Equation (1954) following Kozma & Fransson (1992)
- Determination of contribution to **heat** (thermal energy) and **ionization/excitation** (non-thermal) for all non-LTE atoms/ions/levels
- Treatment of corresponding terms in energy and SEE equations
- On/Off switch in CMFGEN
- Key property: Non-thermal electrons deposit the bulk of their energy as heat



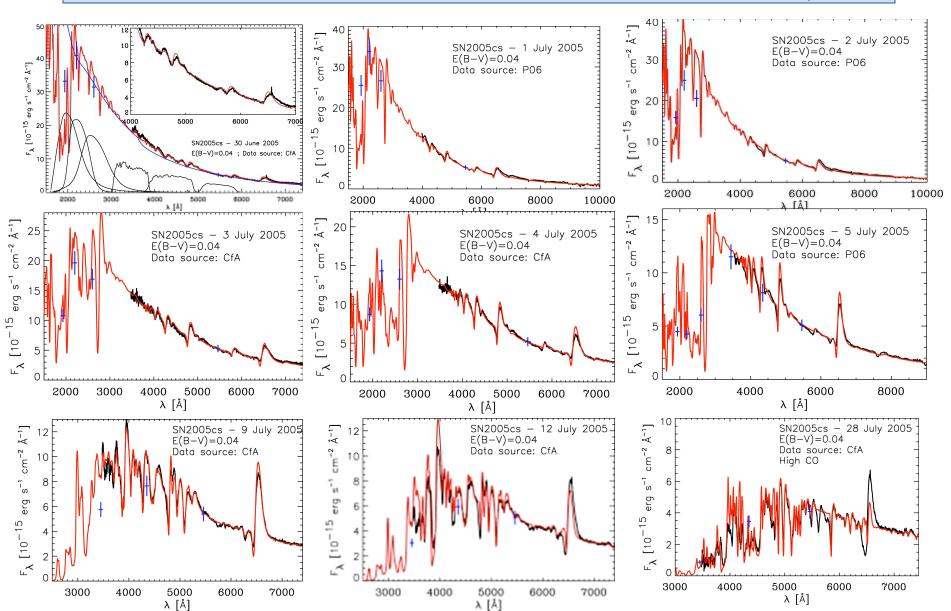
Illustrations with non-LTE Steady-state approach

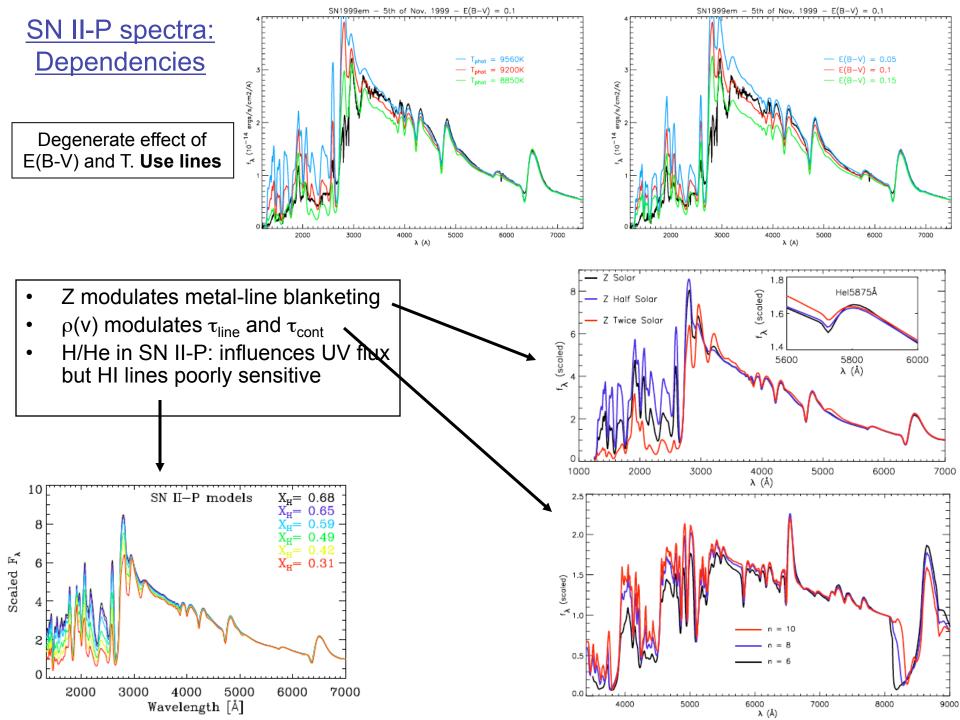
- 1. Application to SNe II-P (SN2005cs)
- 2. Exploration on spectral dependencies

Non-LTE Steady State Approach

Application: Modeling of Young Type II SNe

The case of the Type II-P SN 2005cs in NGC 5194 (Dessart et al. 2008)





1D Non-LTE Time-Dependent Radiative Transfer of Supernova Ejecta with CMFGEN: Results

Dessart & Hillier 2010,2011; Dessart et al. 2011ab

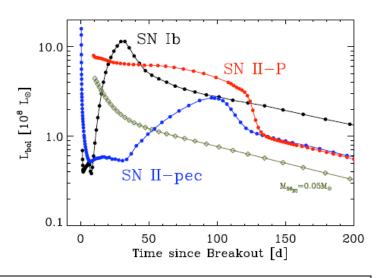
- 1. SN II-pec (87A): evolution from 0.3 to 21d of BSG-progenitor model Im18a7Ad (Woosley, priv. comm.)
- 2. SN II-P: evolution from 10 to 1000d of 1.2B explosion of 15 and $25M_{\odot}$ RSG progenitors.
- 3. SNe Ilb/lb/lc: evolution from 1 to 100d of 1.2B explosion of (quasi) hydrogen-less cores produced from binary-star evolution (Yoon et al. 2010).

Light-curve evolution of SNe II-P, II-pec, Ib: Radiative-transfer simulations on full ejecta using physical input models

Inputs

	pre-SN Star	M∗ [M _☉]	R _∗ [R _⊙]	M _{ejecta} [M _⊙]	E _{expl} [B]	M₅6 _{Ni} [M _☉]
SN II-P	RSG	15 (single)	830	10.9	1.2	0.08
SN II-pec (87A)	BSG	18 (single)	47	15.6	1.2	0.08
SN lb	WN	25 (binary)	10	3.6	1.2	0.24

(DH10,DH11a; Dessart et al. 2011)



- Rapid fading after shock breakout
- Post-breakout plateau: L_{Plateau} function of R_{*}
- Possible re-brightening from ⁵⁶Ni/⁵⁶Co decay. Delay function of mixing.
- High-brightness phase function of M_{*} (large τ), R_{*} (cooling), M_{56Ni} (heating)
- Transition to nebular phase when $\tau \sim 1$; $L_{nebular}$ function of $M_{56}N_{ij}$ and γ -ray trapping
- Not considered: binary (Kasen 2010) or CSM interaction, Magnetar radiation (Maeda et al. 07)

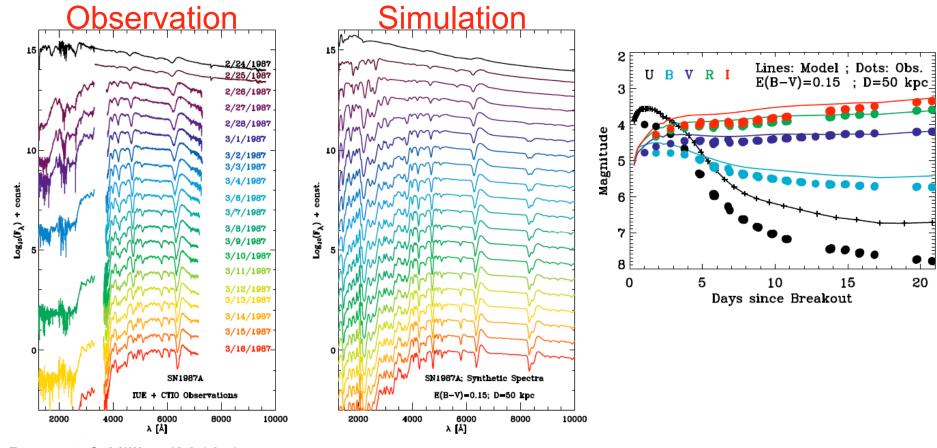
Confrontation of simulations to observations => M*, R*, E_{expl}, ⁵⁶Ni

LCs are degenerate/ambiguous: confusion la/b/c, difference M_{\star} & M_{eiecta} etc.

Comparison to observations of SN1987A

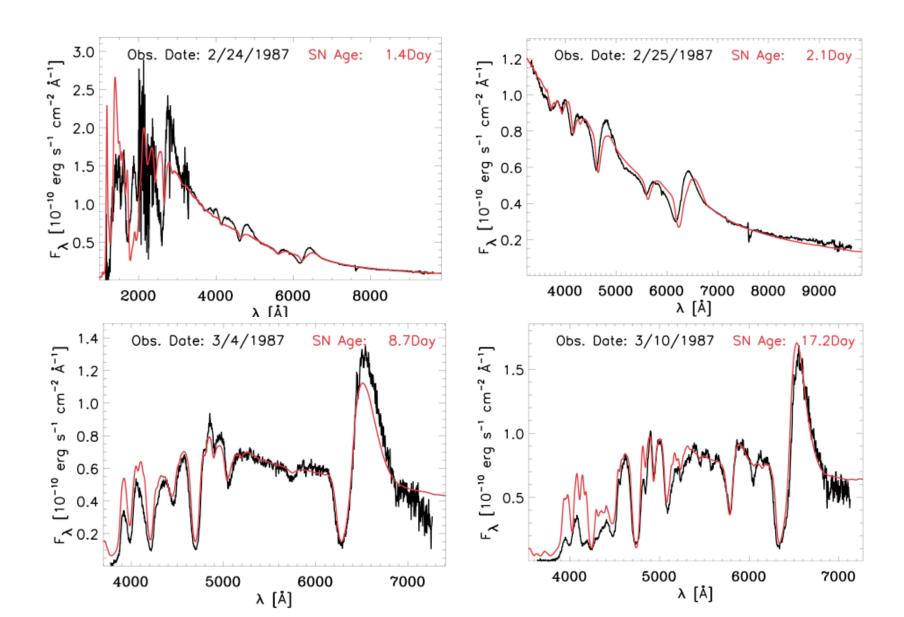
Agreement at 10% level except in the blue (opacity issue)

Supports 18M_☉ BSG progenitor, R_{*}~50R_☉, and E_{expl}~1.2B

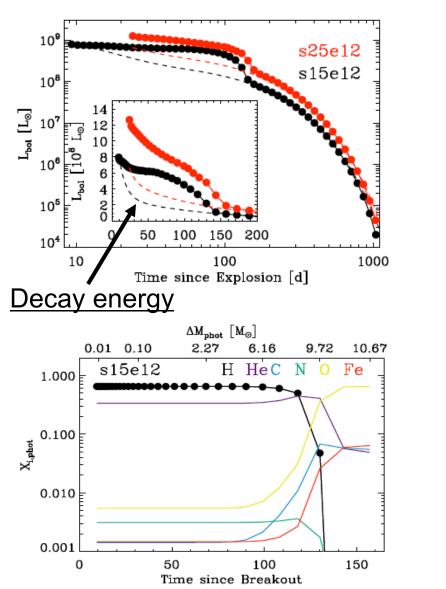


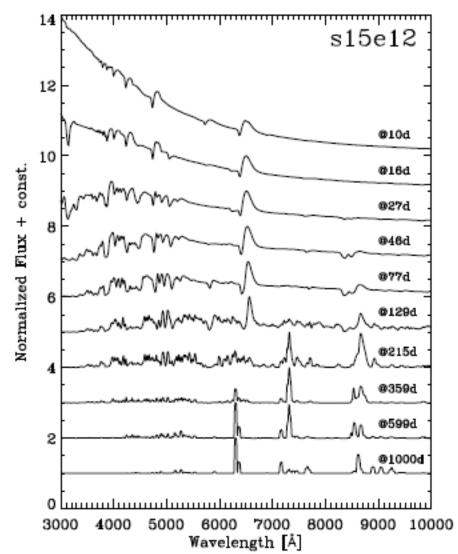
Dessart & Hillier (2010a)

Comparison with SN1987A spectra



Simulations of SNe II-P based on 15 and 25M_☉ progenitor stars *Dessart & Hillier (2011)*

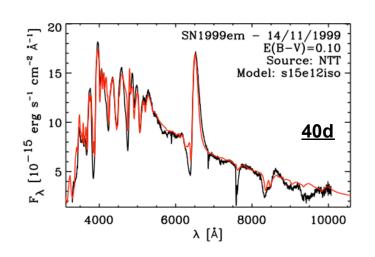




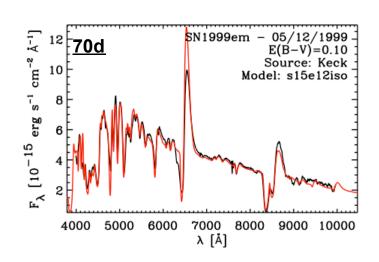
Photospheric phase

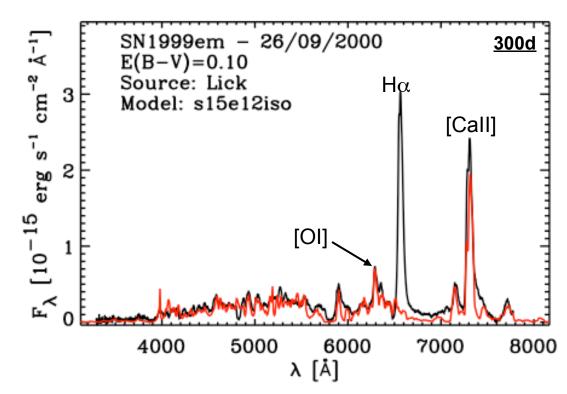
Nebular phase

Comparison to SN 1999em (II-P) spectra

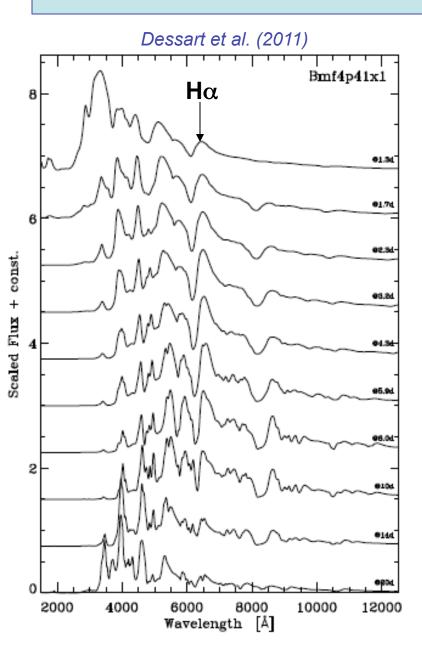


- General agreement at all times
- Specific disagreement with $H\alpha$ at nebular times: neglect of non-thermal processes
- [OI]: important line from He-core oxygen





Simulations of SNe IIb based on binary-star evolution models



- Binary-star progenitor model
- $M_*=18M_{\odot}$, $R_*=12R_{\odot}$; $M_{ejecta}=2.91M_{\odot}$
- $M(He) = 1.63 M_{\odot}, M(H) = 0.006 M_{\odot}$
- X(H) = 0.05 at 50000 km/s
- X(H) < 0.001 at 10000 km/s
- M(⁵⁶Ni)=0.18M_⊙ (unmixed); thermal processes only

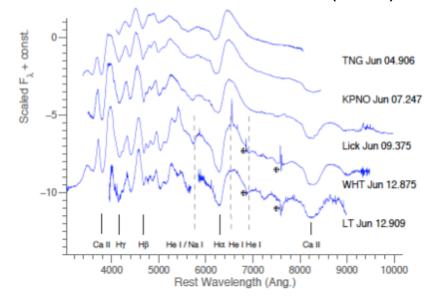
Results

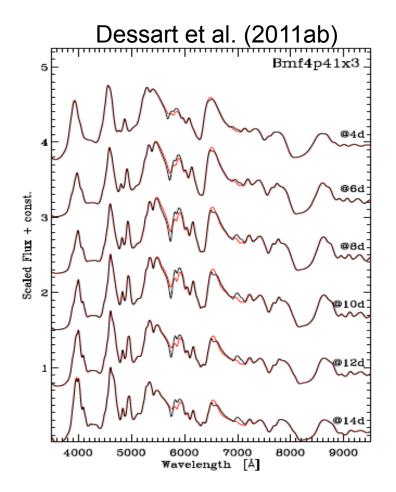
- Hα present for ~20 days, Hel lines for ~10d
- H α present for X(H) as low as 0.01
- Non-LTE time-dependent effects are key
- Non-thermal effects irrelevant/unlikely for H α
- If fast LC + HI lines ⇒ low-mass ejecta & prog.
 because of He-core structure

Comparison with SN IIb 2011dh

Encouraging match to SN 2011dh with SN IIb model 4.4Msun model of Dessart et al. (2011ab) for optical and near-IR ranges.

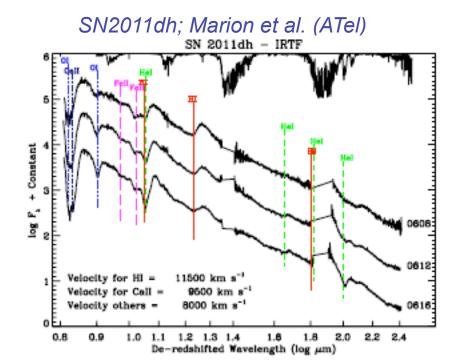
SN2011dh; Arcavi et al (2011)

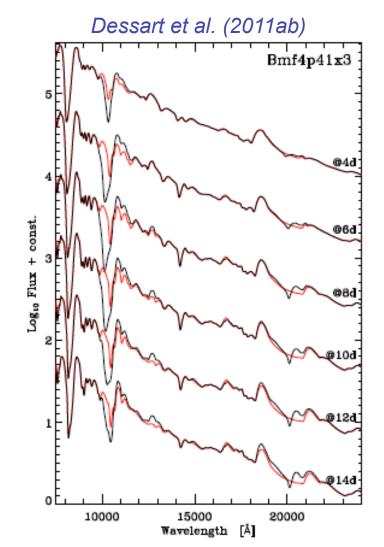




Comparison with SN IIb 2011dh

Encouraging match to SN 2011dh with SN IIb model 4.4Msun model of Dessart et al. (2011ab) for optical and near-IR ranges.



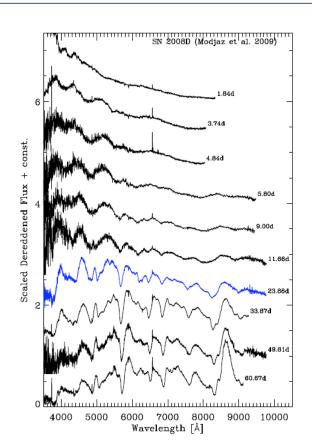


Simulations of SNe Ib based on binary-star evolution models Dessart et al. (in prep)

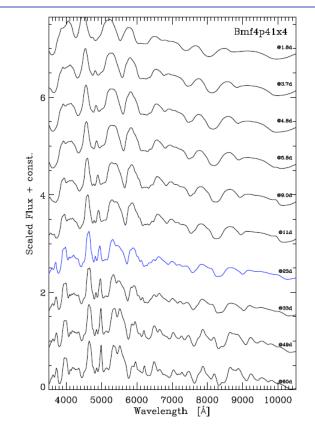
Strongly mixed SN lb model Modeling with non-thermal processes.

SN 2008D:

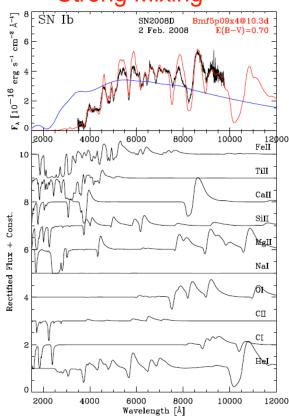
Blue featureless SED (intriguing...), Appearence/persistence of Hel lines, Fast transition to nebular (red SED)



Structured SED
Appearance/persistance of Hel lines
(mixing, non-thermal effects),
Slower transition to nebular (M, ⁵⁶Ni))
SED too blue at late time (⁵⁶Ni)



Strong Mixing

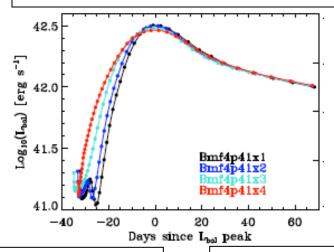


Effect of mixing in Helium-rich ejecta

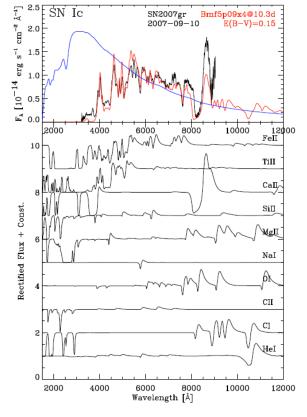
Dessart et al. (in prep)

Models

- 5.1M_☉ **He-rich** pre-SN star model
- 1.2B ejecta with 2 levels of mixing
- Affects ⁵⁶Ni strongly, He weakly
- Non-thermal treatment



Moderate Mixing



Moderate Mixing: SN Ic

Strong Mixing: SN Ib

- No post-breakout plateau
- Prompt rise to peak
- Redder colors and broader spectral lines at peak
- Un-ambiguous presence of numerous Hel lines
- Hel/Fell/Sill in 6000A region but Hel stronger
- Numerous CI/OI/MgII/Call lines at 7000-9000A

- post-breakout plateau.
- Re-brightening delayed
- Bluer colors and narrower spectral lines at peak
- Hel lines present but weak
- Nal/Ol/Hel/Fell/Sill in 6000A region: Messy!
- Numerous CI/OI/MgII/Call lines at 7000-9000A

What distinguishes SN Ib from SN Ic ejecta/progenitors?

Dessart et al. (in prep)

- Binary evolution scenario (Yoon et al. 2010) for production of low-mass lb/c ejecta to match LCs (Ensman & Woosley 1988, Shigeyama et al. 1990, Dessart et al. 2011), rates (Smith et al. 2010).
- Ib versus Ic conditioned by progenitor (mass, composition) and explosion (mixing) properties.

SNIb

- $X_{56_{Ni}} \ge 0.01$ at **He-rich** photosphere to yield Hel lines \Rightarrow Hard to make a SN lb
- High X_{56Ni} ⇒ SNe Ib likely from lower-mass progenitors (small cores) + efficient mixing
- SN Ib progenitors: low-mass "WR" in binaries (weak winds, low L)? Undetected by current WR searches. Associated with a massive star companion at explosion.

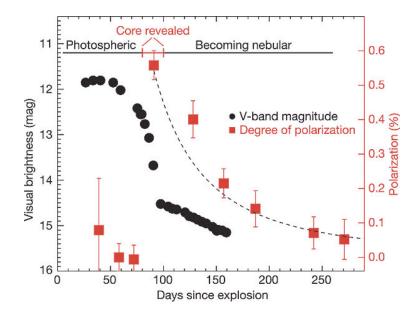
SNIc

SNe Ic likely from **higher mass progenitors** (larger O buffer between He and ⁵⁶Ni) or ejecta that are **He deficient or insufficiently mixed**. SNe Ic easier to make.

Spectro-polarimetry: Explore departures from spherical symmetry

- Inputs from non-LTE time-dependent SN II-P simulations (DH11a)
- Fixed imposed asymmetry
- Polarization due to electron scattering
- Plateau-phase: Low-polarization due to strong cancellation effects, optical depth effects, steep mass-density and/or electron-density distribution
- For a fixed asymmetry, polarization jumps at onset of nebular phase
- Nebular-phase: polarization commensurate to magnitude of asymmetry

Leonard et al. (2006) - Type IIP SN2004dj

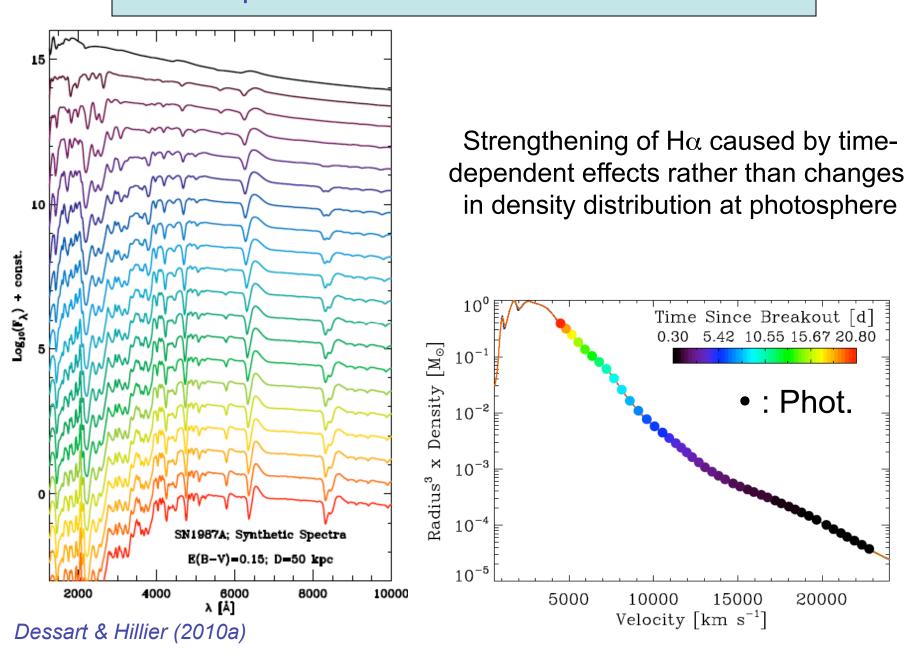


Days since End of Plateau

Summary

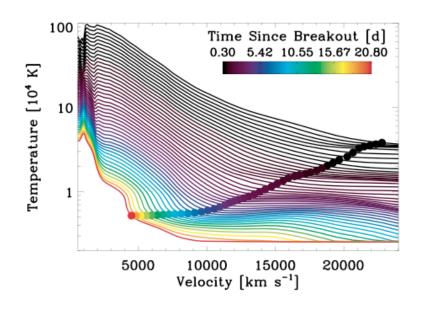
- Original goals met. CMFGEN 'works' for all SN types, fully non-LTE, fully timedependent, treatment of non-thermal processes, local/non-local energy deposition
- Code benchmarked against 87A (!) and SNe II-Plateau observations.
- Non-LTE and steady state adequate at early times in Type II SN. Allows "good fits" through tuning parameters. Good for distance determinations with Type II SNe. Dependencies.
- Late-time modeling requires time-dependent treatment (e.g. Dn/Dt) => Ionization freeze-out
- Non-thermal processes key in low-ionized conditions (SNe II, Ib, Ic).
- Higher physical consistency with "Full-Ejecta" simulations based on hydrodynamical inputs
 of pre-SN evolution and explosion. NO artificial inner-boundary condition.
- Versatile tool for simultaneous spectroscopic and light-curve modeling of SNe.
- Great tool when combined with radiation-hydrodynamics (hydro inputs) and stellar evolution models (pre-SN star)
- Gear-up for current/forthcoming blind surveys: PTF, PAN-STARRS, Sky-Mapper, and LSST

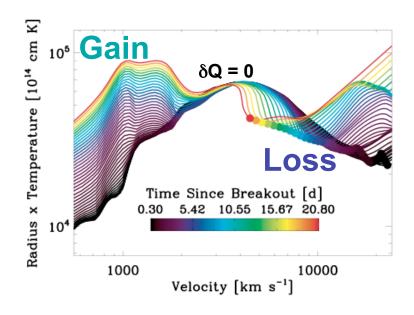
Comparison to observations of SN1987A



SN1987A Full-Ejecta Evolution

Global cooling due to expansion: T ∝1/R
Compensated by decay at depth
Exacerbated by radiative losses at surface





SN1987A Ejecta Evolution

- Electron density N_e set by rate equations and charge conservation (neutrality)
- Mass continuity equation + homologous expansion => Density

 1/R³
- Recombination => N_e drops faster than 1/R³ (Lagrangian sense)

